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## **Машинное обучение. Введение в генеративные модели. VAE.**

Основная идея, стоящая за генеративными моделями – понять, как устроен изучаемый процесс, воспроизводя его. Имея обученную генеративную модель, мы можем создавать вещи с нуля. Сегодня такие модели широко используются для создания картинок, рукописного текста, синтеза речи.

## **Machine Learning. Introduction to Generative Models. VAE.**

### *Introduction*

Machine Learning is a scientific discipline that answers the following question: How computer programs can automatically learn and make intelligent decisions based on data.

In 1946 the first computer system ENIAC was developed. In 1950 Alan Turing proposed a test to measure the ability of a machine to learn. The Turing test is based on the idea that we can only determine if a machine can actually learn if we communicate with it and cannot distinguish it from another human. For today no machine has completed the entire test.

Last decade machine learning received new boost from gathering huge amount of statistical data and scaling up computations, particularly GPU.

Accumulated information describes processes that surround us. One practical problem is to teach a machine the process using information we already have. In order to understand the process we try to create a model reproducing it. This model is named generative. Researches see huge potential and expect to get semantic and interpretable features of natural processes. This article will introduce one of these models.

The main idea behind generative models is “what I cannot create I cannot understand”. Having trained generative model we can create things from scratch. Today popular models are used to generate real looking images, handwriting text, and speech.

#### Variational Auto encoders

In the tasks of statistical learning getting generative model is the most desirable result and at the same time it is most challenging. Frequently it is excessive. Consider generating natural images as an example of learning generative model task and start from formal objective setting.

It is possible to distinguish between two formulations of the problem to recover joint probability function. They are complete-data and incomplete-data recovering. The second one is more general and of more interest to us. Today the popular algorithm to obtain generative model for incomplete-data case is Variational Auto encoders (VAE) that was introduced by Max Welling and Kingma in 2015. The example of algorithm to deal with complete-data is GAN (Generative Adversarial Networks)

In this article we explain the core principles of the design and learning VAE model. Then we will conclude about strengths and weaknesses of VAE. For the sake of clarity, we make explanation using an example of generating images of handwritten digits. We will use MNIST that is a popular dataset for our experiments.

Let we have some dataset  $X = \{x^{(i)}\}_{i=1}^N$  consisting of  $N$  independent samples of some random variable  $x$ . Every  $x$  from  $X$  is the image of a digit presented by a vector of pixels. Each pixel represented by number, 0 for white and 1 for black. We should note that all  $x$  from  $X$  are independent and identically distributed. Every sample  $x$  from  $X$  is placed in correspondence with value of random variable  $z$ . The values of variable  $z$  we cannot observe. For example,  $z$  can encode the digit presented on the image or font characteristics. Likewise  $z$  is named latent variable or silent variable.

We consider that the process of painting is arranged as follows:

- We sample  $z$  from according prior distribution  $p_\theta(z)$ . We assume that this distribution comes from parametric family and we know it up to parameter  $\theta$ .

- Then we sample  $x$  according to conditional distribution  $p_\theta(x | z)$  using  $z$  we have chosen on the first step. This distribution also comes from parametric family and we know it up to parameters

$$p_\theta(x, z) = p_\theta(z) \cdot p_\theta(x | z)$$

For example, first choose digit from 0-9, italic font and then draw. Our goal is having dataset  $X$  to find  $\theta^*$  that maximize the logarithm of likelihood to have a chance to observe  $X$ .

$$\theta^* = \operatorname{argmax}_\theta \sum_i^N \log p_\theta(x^{(i)})$$

Actually in this formulation we can apply EM algorithm. However it becomes difficult or even impossible when an intractable integral arises during computing posterior probability density function (PDF). This can happen in case of modelling  $p_\theta(x | z)$  with neural network with nonlinear hidden layer.

$$p_\theta(x) = \int_{z \in Z} p_\theta(z) \cdot p_\theta(x | z) dz$$

$$p_\theta(z | x) = \frac{p_\theta(x | z) \cdot p_\theta(z)}{p_\theta(x)}$$

VAE algorithm was invented to solve this difficulty. VAE as well as EM is trying to maximize the lower bound of log likelihood. The core idea of VAE is parametrization trick to solve problem of sampling from  $q_\phi(z | x)$

If we have random variable  $z$  from some distribution  $p(z)$ , we often can replace  $z$  with differentiable mapping  $g$  of another random variable  $z = g_\phi(\epsilon)$ , where  $\epsilon$  is random variable with its own distribution. All comes down to choosing  $g$  and parameters  $\phi$ . The values of parameters can be found during learning process because we choose  $g$  to be differentiable with respect to parameters.

$$z = g_\phi(\epsilon), \text{ where } \epsilon \text{ is random variable}$$

In our case we'd like to apply this trick to approximate posterior

$$z \sim p(z | x)$$

$$z = g_\phi(\epsilon, x)$$

The parametrization trick allows us to rewrite an expectation with respect to  $q_\phi(z|x)$  to make its Monte Carlo estimate differentiable w.r.t  $\phi$ . What eventually allows to perform optimization of lower bound more effectively. We can emphasize the following features of VAE algorithm:

1. Easy to train and get working.
2. Allows to explicitly obtain generative model.
3. Allows to obtain discriminative model.
4. Works with incomplete-data.
5. Allows to put in model hypothesis about prior distribution of latent variables
6. Not always possible to get interpretable representations of model parameters
7. Generative model output is blurred

Nowadays there exist improvements of VAE. One of them is Variational Inference with Inverse Autoregressive Flow was introduced by Durk Kingma and Tim Salimans. The idea is the transformation of a simple distribution over the latent variables into a much more flexible distribution. This allows choosing arbitrary approximate posterior distribution and making parallel computations.

Another one is a combining GAN and VAE [1] model named adversarial auto encoders to get benefits of each of them

### *Experiments*

We trained VAE model using images from MNIST dataset. Figure 1 depicts the manifold of digits the trained model can produce. We optimized model with two dimensional latent space using ADAM - improved method of gradient descent.

To get the picture we transform linear spaced coordinates from unit square to Gaussian space to get samples of variable  $z$ . Then for each  $z$  we obtain  $p_\theta(x|z)$  using  $\theta$  we learned during training. We used mini batches of size 100 and made  $1e6$  steps.

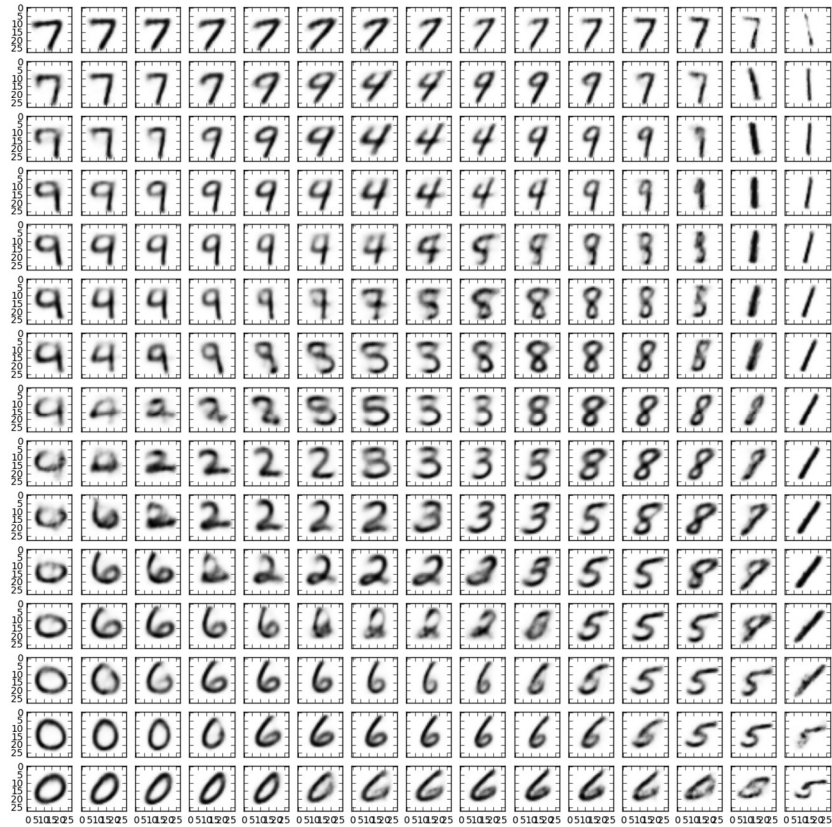


Fig. 1. MNIST manifold

### *Ideas of practical usage of generative models*

One of the most desirable applications of generative models is extracting interpretable and disentangled representation of parameters of model. Using digit generation example we would like to have the parameter representing the value of digit on the picture. Put in other word we would like to have a non-deterministic mapping that takes number from 0 to 9 and produces the image of this number. In general we want to build mappings taking names of objects to draw. Similar we could generate sounds, voice, text and other things created by nature or human.

Today generative models are used for image noise reduction, super-resolutions, recover lost information on images or videos, neural network pretraining.

The accumulated data from various areas of human life are waiting to be processed to reveal new knowledge.

## **Список литературы:**

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## **V. Спорт и здоровьесберегающие технологии**

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### **Анализ результативности в беговых дисциплинах легкоатлетов Свердловской области на чемпионатах России в постсоветский период**

Статья посвящена анализу результативности выступлений легкоатлетов Свердловской области в беговых дисциплинах на Чемпионатах России в постсоветский период. На основе анализа динамики выступлений легкоатлетов Свердловской области сделаны выводы об успешности выступлений легкоатлетов Свердловской области.